UNIT 5: Rules Integration / Integration Concepts Highlights

As a Concept: Integration is referred to as the "anti-derivative" or the inverse process of the finding the derivative. It provides us a way to get from a function calculating change back to the original.

An indefinite integral refers to an integral in its general form that represents a family of functions.

$$\int f(x)dx = F(x) + C$$

Basic Integration Rules

$\int x^n dx$	→	$y = \frac{x^{n+1}}{n+1} + C$	$\int v(t)dt$	→	$s(t) + C$
$\int \frac{1}{x} dx$	→	$y = \ln\lvert x\rvert + C$	$\int a(t)dt$	→	$v(t) + C$
$\int e^x dx$	→	$y = e^x + C$	$\int a^x dx$	→	$y = \frac{a^x}{\ln a} + C$

Trigonometric Integration Rules

- ✓ Basic Trig Integrals

Function	Antiderivative
$f(x) = \sin x$ or $\int \sin x\, dx$	$F(x) = -\cos x + C$
$f(x) = \cos x$ or $\int \cos x\, dx$	$F(x) = \sin x + C$
$f(x) = \sec^2 x$ or $\int \sec^2 x\, dx$	$F(x) = \tan x + C$
$f(x) = \csc^2 x$ or $\int \csc^2 x\, dx$	$F(x) = -\cot x + C$
$f(x) = \sec x \tan x$ or $\int \sec x \tan x\, dx$	$F(x) = \sec x + C$
$f(x) = \csc x \cot x$ or $\int \csc x \cot x\, dx$	$F(x) = -\csc x + C$

- ✓ Inverse Trig

$$\int \frac{f'(x)}{\sqrt{1-[f(x)]^2}} dx \quad \rightarrow \quad y = \sin^{-1} f(x) + C$$

$$\int \frac{f'(x)}{1+[f(x)]^2} dx \quad \rightarrow \quad y = \tan^{-1} f(x) + C$$

Eliminating the Constant (+C)

When given initial condition, the value of the constant (+C) can be determined after the integration by substituting the given into the result.

- ✓ Working backwards from f'' with initial conditions
- ✓ Working backwards from $a(t)$ with initial conditions

Ex: $f''(x) = 24x^2 - 18x + 2, \quad f'(1) = 6 \quad and \quad f(1) = 3$

$$f'(x) = 8x^3 - 9x^2 + 2x + 5$$
$$f(x) = 2x^4 - 3x^3 + x^2 + 5x - 2$$

Integrating with U-Substitution

✓ Algebraic (Quantity raised to power)

Procedure:

1. Let "u" equal the algebraic quantity on the inside.
2. Find $\frac{du}{dx}$.
3. Match $\frac{du}{dx}$ to the original function.
4. Substitute.
5. Find the antiderivative in terms of "u".
6. Re-substitute for u in terms of x.

$$\int x^3(x^4+3)^2\,dx = \frac{(x^4+3)^3}{12}+C$$

✓ Trigonometric (Quantity raised to a Power)

Procedure:

1. Let "u" equal the trigonometric ratio being raised to the power (inside quantity).
2. Find $\frac{du}{dx}$.
3. Match $\frac{du}{dx}$ to the original function.
4. Substitute.
5. Find the antiderivative in terms of "u".
6. Re-substitute for u in terms of x.

$$\int \sin^6 x\cos x\,dx = \frac{\sin^7 x}{7}+C$$

✓ Trigonometric (Unusual Angle)

Procedure:

1. Let "u" equal the unusual angle.
2. Find $\frac{du}{dx}$.
3. Match $\frac{du}{dx}$ to the original function.
4. Substitute.
5. Find the antiderivative in terms for trig(u).
6. Re-substitute for u in terms of x.

$$\int 2x\cos(x^2+3)\,dx = \sin(x^2+3)+C$$

✓ Exponential

Procedure:

1. Let "u" equal the exponent
2. Follow previous procedures using u.

$$\int 4xe^{x^2}\,dx = 2e^{x^2}+C \qquad \int 2^{3x}\,dx = \frac{2^{3x}}{3\ln 2}+C$$

✓ Logarithmic

When handling the integral of a rational expression, look for the situation where the numerator is being expressed as the derivative of the expression in the denominator" (or can be manipulated into that form).

Procedure:

1. Let "u" equal the denominator.
2. Follow previous procedures using u.

$$\int \frac{x+2}{3x^2+12x}\,dx = \frac{1}{6}\ln\left|3x^2+12x\right|+C$$

Evaluating Definite Integrals

A definite integral refers to an integral that has a lower limit and an upper limit which can be evaluated to a specific value.

$$\int_a^b f(x)dx = [F(b)+C]-[F(a)+C] = F(b)-F(a)$$

*[Upper – Lower]!

Properties of Definite Integrals

I. Addition Property:

If $a < b < c$, then $\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$

II. Coefficient Property:

For any Real Number *c*, $\int_a^b cf(x)dx = c\int_a^b f(x)dx$

III. Bounds Property:

$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

IV. Integral Sum/Difference Property:

$$\int_a^b (f(x) \pm g(x))dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$$

Separation of Variables (Differential Equations)

Procedure:

1. Use differentials to get like variables on the same side.
2. Find the antiderivative of each side.
3. Put into *y* – form, if possible.

$$\frac{dy}{dx} = \frac{-2x}{y^2} \quad \rightarrow \quad y = \sqrt[3]{-3x^2 + C}$$

Integration Concepts (Theorems/Connections)

✓ Fundamental Theorem of Calculus (FTOC) (pg. 386)

-If f is a continuous function on [a,b] and x is a point in (a,b), then $\frac{d}{dx}[\int_a^x f(t)dt] = f(x)$

This theorem serves as the link between Differentiation and Integration as inverse processes. The statement below says that "if f is integrated and the result is differentiated, we are back at the original function".

✓ Total Change Theorem (pg. 377)

$$\int_a^b F'(x)dx = F(b) - F(a)$$

We know that $F'(x)$ represents the rate of change of y = F(x) with respect to x, and [F(b) - F(a)] represents the change in y as x changes from a to b, so this can be constructed to form what is called the Total Change Thm.

✓ Average Value definition (pg. 470)

Average Value = $\frac{1}{(b-a)}\int_a^b f(x)dx$

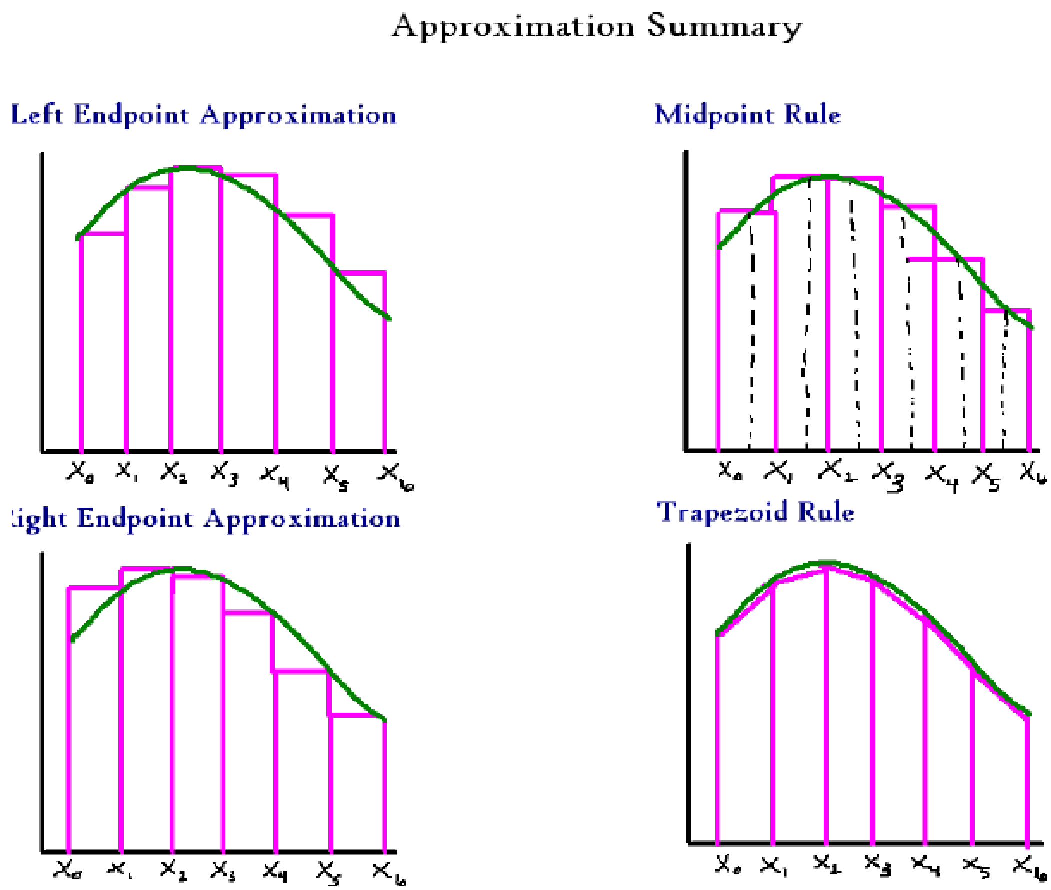
This calculates the avg value of the function on the stated interval [a,b].

✓ Mean Value Theorem (pg. 470)

$\int_a^b f(x)dx = f(c)(b-a)$ OR f(c) = average value of f(x)

This will find the specific value on [a,b] where the average value of the function is being attained.

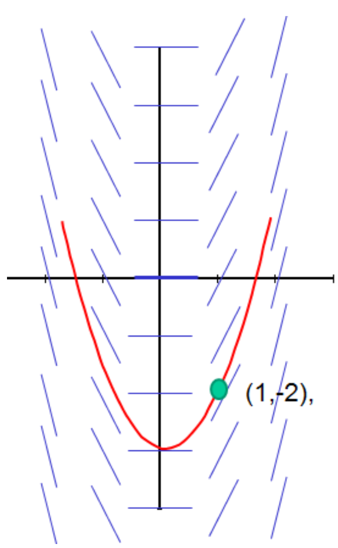
Approximation Methods involving Summation



NOTE: Most commonly will be applied to draw conclusions from **<u>a table OR graph of data</u>** where the function is unknown.

Slope Fields

A slope field is a tool that can be used to approximate the graph of a function when you know the derivative. If given an initial condition, you will then be able to sketch the original function.



$$y' = 2x$$

If you know an initial condition, such as (1,-2), you can sketch the curve.

By following the slope field, you get a rough picture of what the curve looks like.

In this case, it is a parabola.

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